Resit exam
SOLID MECHANICS (NASM)
February 4, 2022, 19:45-21:45 h

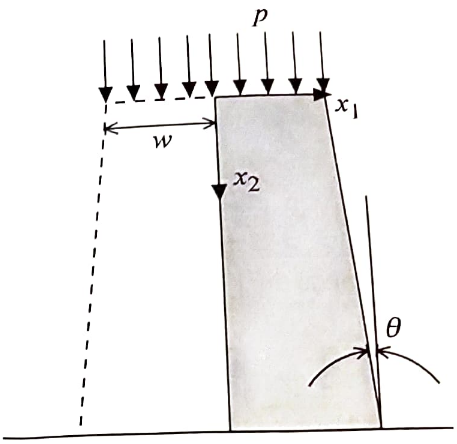
This exam comprises three problems, for which one can obtain the following points:

Question	# points
1	1 + 1 + 2 + 1 = 5
2	2 + 1 + 1 = 4
3	1.5 + 1 = 2.5

The number of points is indicated next to each subquestion inside a rectangular box in the right-hand margin on the next pages.

The exam grade is calculated as $9 * (\#\text{ points})/11.5 + 1$. The final grade for this course is also based on the grade for the ComSol report.

Question 1 A long ridge is cut out of a single crystal by means of focused ion beam milling. Because of this process the sides of the ridge are not perfectly vertical but have a small taper angle θ. The properties of the crystal are tested by subjecting the top to a pressure, p, which is assumed to be uniformly distributed over the top width w. Since the ridge is long, we can also assume that it deforms in plane strain perpendicular to the x_1–x_2 plane shown. Because of symmetry only half of the cross-section (the gray area) needs to be analyzed.



a. Specify the boundary conditions for the left and the right-hand side of the gray region in terms of stress components. [1]

b. Assuming the vertical stress σ_{22} to be uniform in x_1, determine the value at arbitrary depth x_2. [1]

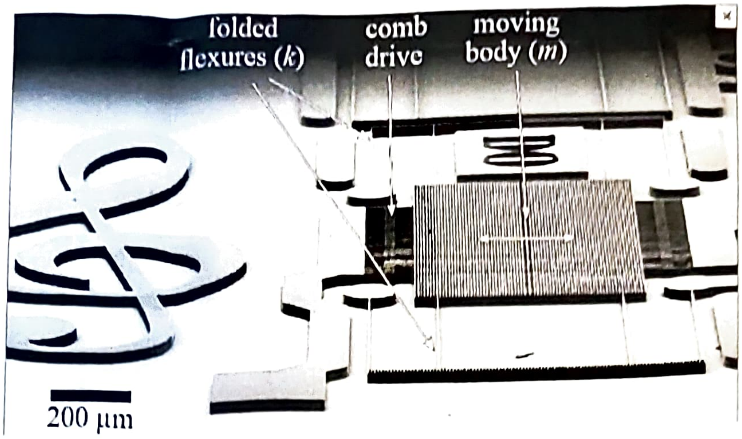
c. Use equilibrium to find the differential equation for the shear stress σ_{12} in terms of σ_{22}. Solve this equation, taking into account the appropriate boundary conditions, and prove that the stress field satisfies

$$\frac{\sigma_{12}}{\sigma_{22}} = \frac{\xi_1}{1+\xi_2}, \quad \text{with } \xi_1 := (x_1/w)\tan\theta\,, \xi_2 := (x_2/w)\tan\theta$$

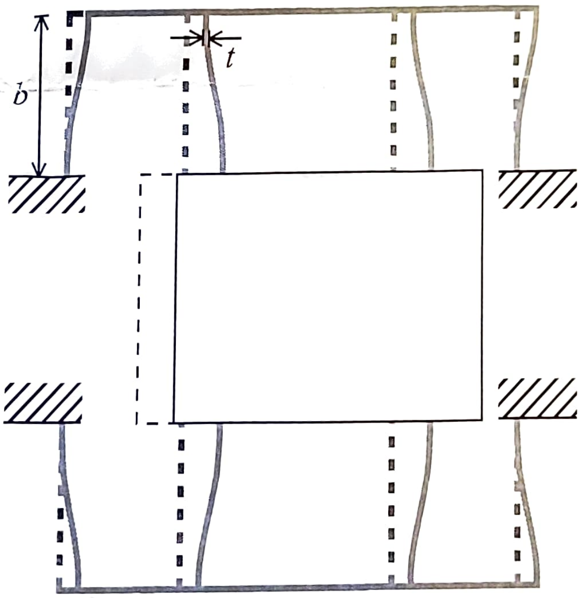
[2]

d. In the same spirit, you could develop the differential equation for σ_{11}. You do not have to perform the actual derivation, but (i) point out which boundary condition you would use to solve the differential equation and (ii) demonstrate that σ_{11} varies quadratically with x_1. [1]

Question 2

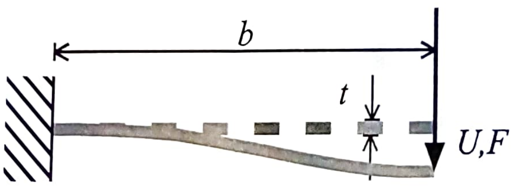


About a decade ago, a group of nanotechnology students from Twente University presented a musical instrument called a "micronium". It is special because it is able, by mechanical means, to produce tones with audible frequencies, despite the fact that the vibrating mass weighs only a few dozen micrograms. This is non-trivial in view of the well-known formula $f = \sqrt{k/m}/(2\pi)$ for the eigenfrequency of a mass (m)–spring system (stiffness k): at small scales, the mass m is necessarily small so that the stiffness needs to be small in order to keep f in the audible range ($\sim$ 20–20.000 Hz). The amplitude of the vibrations is only a few micrometers. The device has a separate unit for each tone; a scanning electron microscopy image of one such unit is shown in the figure above. It consists of a thin rectangular plate that is mounted in between spring structures at the top and the bottom consisting of thin slender beams (the comb structures on the left and the right-hand side serve to actuate ('pluck') the mass-spring system and as a sensor to pick up of the frequency, which is then sent to an amplifier; any possible influence of the comb structures can be ignored here).



The figure on the right-hand side shows a mechanical model, comprising four interconnected leaf springs on top and bottom, having length b and width t (the thickness of the springs and the rectangular mass are the same). Both the equilibrium position (dashed lines) and a deflected configuration are shown (the outer leaf springs are mounted on posts fixed to the wafer, the hatched regions in the schematic).

a. First determine the stiffness F/U of a single leaf spring against a sideway displacement U, see figure below.

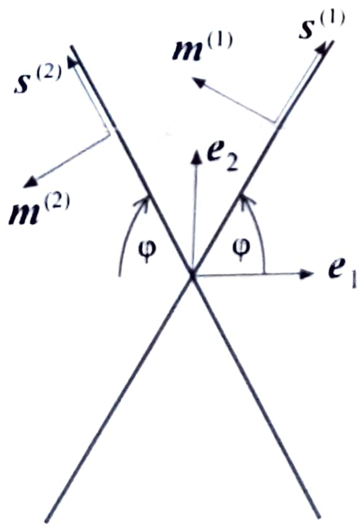


Note: this can be done either by solving the differential beam equation (3.49) plus appropriate boundary conditions or by combining the *Mysotis Mechanicus* of Fig. 3.6 in a clever way. [2]

b. Denoting the stiffness determined above by k_1, compute the total stiffness k of the spring system in a micronium unit. [1]

c. Using values of Young's modulus $E = 112$GPa and mass density $\rho = 2.3\mathrm{g/cm}^3$ for silicon, estimate the thickness t of the springs in order that the frequency of the tone produced by the micronium unit is 1000Hz. Note: from the micrograph at the start of this question, one could derive that $b = 200\ \mu$m and that the moving mass has an area of $500 \times 600 = 30 \times 10^4\ \mu\mathrm{m}^2$. [0.5]

Question 3 A crystal is oriented such that there are two slip systems, $\left(\boldsymbol{s}^{(1)}, \boldsymbol{m}^{(1)}\right)$ and $\left(\boldsymbol{s}^{(2)}, \boldsymbol{m}^{(2)}\right)$, oriented symmetrically with respect to the $\boldsymbol{e}_1$-direction by an angle φ, as shown in the figure below. Consider the situation where the slips on both systems are equal, $\gamma^{(1)} = \gamma^{(2)}$.



a. Determine the principal directions as well as the principal values of the plastic strain tensor. [1]

b. What is the stress state (as a tensor) that could have led to such a plastic deformation? Is this state unique? [1]